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SSIM REVISITED: FROM STRUCTURAL SIMILARITY TO ERROR VISIBIITY

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Outline

- **Introduction**
- **Alternative model for computing SSIM**
- **Advantages**
  - Methodological – new reasoning explaining why SSIM works
  - Encoding optimizations for boosting SSIM scores
  - Relationship between SSIM and other objective metrics
  - Path toward improvements and extensions
- **Experimental validations**
- **Conclusions**

INTRODUCTION

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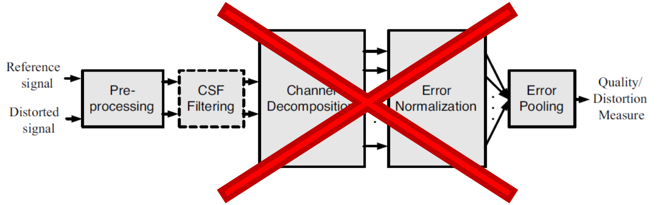
SSIM concept

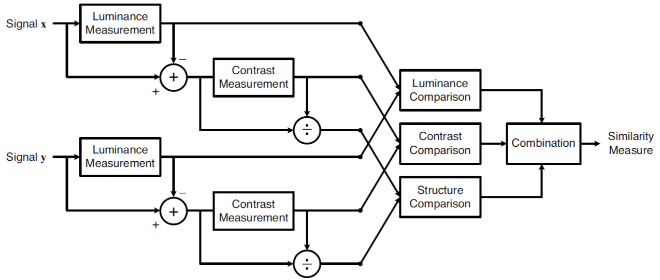
SSIM concept (Wang et al 2004)

- Build metric that is simpler than multi-band CSF-based metrics
- Instead of modeling vision, measure “structural similarity” between images



Original SSIM definition:

- Patch-based computation (11x11 patches with Gaussian weights/ σ=1.5)
- A product of 3 quantities:
  - differences in luminance: $l(x,y) = \frac{2\mu_x\mu_y + C1}{\mu_x^2 + \mu_y^2 + C1}$
  - differences in contrast: $c(x,y) = \frac{2\sigma_x\sigma_y + C2}{\sigma_x^2 + \sigma_y^2 + C2}$
  - differences in structure: $s(x,y) = \frac{\sigma_{xy} + C2}{\sigma_x\sigma_y + C2}$
- Frame-level SSIM = aggregate of patch-level estimates.



$$SSIM(x,y) = l(x,y) \cdot c(x,y) \cdot s(x,y)$$
$$= \frac{2\mu_x\mu_y + C1}{\mu_x^2 + \mu_y^2 + C1} \cdot \frac{2\sigma_x\cancel{\sigma_y} + C2}{\sigma_x^2 + \sigma_y^2 + C2} \cdot \frac{\sigma_{xy} + C2}{\sigma_x\cancel{\sigma_y} + C2}$$
$$= \frac{2\mu_x\mu_y + C1}{\mu_x^2 + \mu_y^2 + C1} \cdot \frac{2\sigma_{xy} + C2}{\sigma_x^2 + \sigma_y^2 + C2}$$

Some questions:

- With default C1,C2 constants the second part of SSIM reduces to: $\frac{2\sigma_{xy}}{\sigma_x^2 + \sigma_y^2}$!
  - This is no longer a measure of difference in contrast or structure!
  - What it measures then?
- It was suggested [Horé & Ziou 2010] that: $\frac{1-SSIM}{SSIM} \sim \frac{MSE}{2\sigma_{xy}}$
  - Is this true? Is there a real connection between SSIM and MSE/PSNR?

Some References

1. Z. Wang, A. Bovik, H. Sheikh, E. Simoncelli, "Image quality assessment: from error visibility to structural similarity". IEEE Transactions on Image Processing, vol. 13, no. 4, pp. 600–612, 2004.
2. Z. Wang, E. P. Simoncelli and A. C. Bovik, "Multiscale structural similarity for image quality assessment," 37th Asilomar Conference on Signals, Systems & Computers, 2003, vol.2, pp. 1398-1402.
3. R. Dosselmann and X. D. Yang, "Existing and emerging image quality metrics," Proceedings of the Canadian Conference on Electrical and Computer Engineering, pp.1906-1913, 2006.
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ALTERNATIVE MODEL OF SSIM COMPUTATION

Proposed Alternative Model

Symmetric 2-band definition

- Full formula:

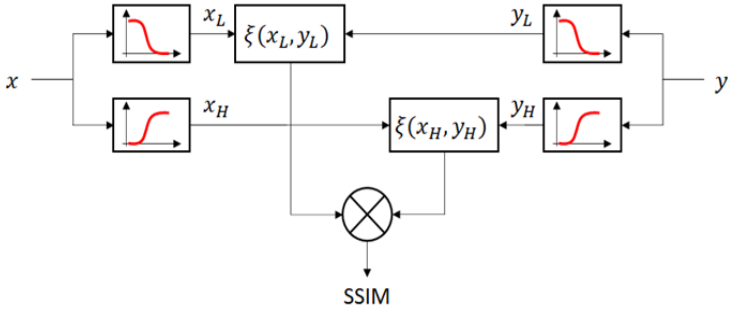
$$SSIM(x,y) \approx \xi(x_L, y_L) \cdot \xi(x_H, y_H)$$

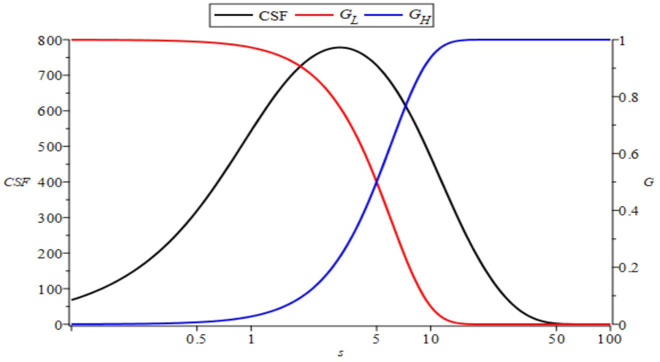
- Where

$$\xi(x,y) = \frac{2\mathbf{E}[xy]+C}{\mathbf{E}[x^2]+\mathbf{E}[y^2]+C}$$

are identical operators applied to low-pass and high-pass versions of images x, y.

Processing diagram:



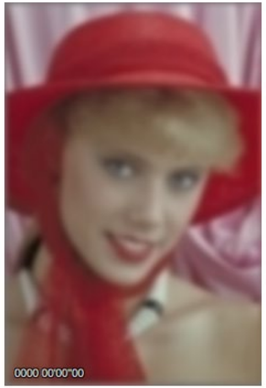


Band Separation:

- Low-pass and high-pass are obtained by Gaussian filter with sigma=3.
- In viewing settings where display Nyquist is 40 cpd this implies split at point of peak sensitivity of human vision.

Example of operation

x



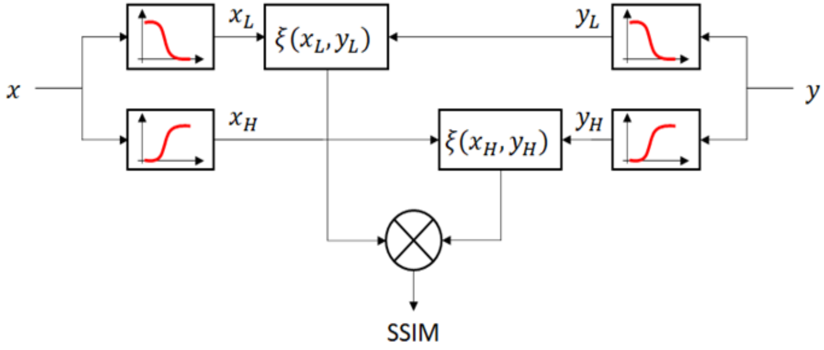
x_L

x_H

y

y_L

y_H



$x - y$

$x_L - y_L$

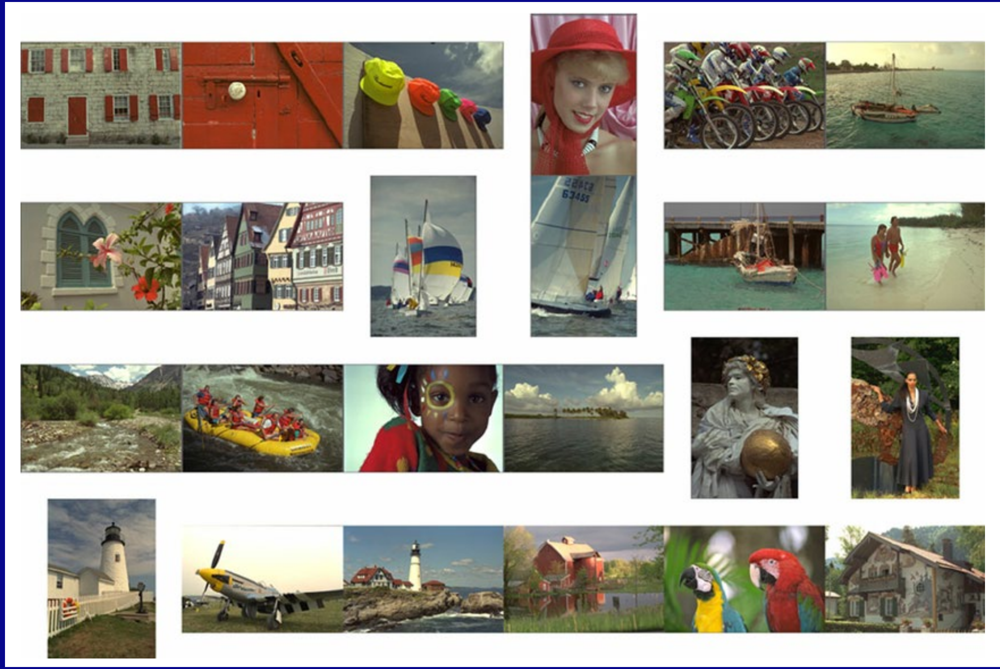
$x_H - y_H$

Experimental Validations

Impairments:

- Codec introduced noise
- Resolution changes
- Gaussian blur
- Additive noise

Data set: Kodak test images



Codec noise / hi-res images

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TABLE I. ACCURACY OF THE PROPOSED METHOD FOR COMPUTING SSIM: H.264 ENCODING/DECODING, 1536X1024 IMAGES..

File	QP=17		QP=22		QP=27		QP=32		QP=37		QP=42		QP=47	
	SSIM	Delta	SSIM	Delta	SSIM	Delta	SSIM	Delta	SSIM	Delta	SSIM	Delta	SSIM	Delta
k01	0.9949	0.0003	0.9867	0.0008	0.9697	0.0016	0.9337	0.0028	0.8687	0.0038	0.7766	0.0026	0.6648	-0.0026
k02	0.9928	0.0000	0.9784	0.0001	0.9447	0.0001	0.8962	-0.0009	0.8318	-0.0033	0.7569	-0.0069	0.6843	-0.0119
k03	0.9879	0.0000	0.9768	-0.0001	0.9669	-0.0003	0.9506	-0.0008	0.9251	-0.0016	0.8947	-0.0033	0.8617	-0.0059
k05	0.9906	-0.0001	0.9782	-0.0004	0.9603	-0.0009	0.9327	-0.0018	0.8937	-0.0034	0.8453	-0.0063	0.7928	-0.0108
k06	0.9941	0.0003	0.9869	0.0005	0.9757	0.0007	0.9551	0.0008	0.9168	0.0004	0.8552	-0.0015	0.7619	-0.0067
k07	0.9930	0.0001	0.9851	0.0003	0.9709	0.0006	0.9427	0.0006	0.8920	0.0000	0.8126	-0.0027	0.7108	-0.0076
k08	0.9889	-0.0002	0.9781	-0.0004	0.9707	-0.0008	0.9599	-0.0016	0.9427	-0.0030	0.9173	-0.0056	0.8811	-0.0087
k11	0.9956	0.0003	0.9853	0.0010	0.9626	0.0022	0.9326	0.0030	0.8876	0.0030	0.8180	0.0019	0.7076	-0.0030
k12	0.9893	0.0000	0.9623	0.0001	0.9320	-0.0001	0.9180	-0.0006	0.8989	-0.0016	0.8708	-0.0038	0.8376	-0.0065
k13	0.9898	0.0000	0.9645	0.0001	0.9328	-0.0004	0.9149	-0.0014	0.8931	-0.0031	0.8649	-0.0055	0.8321	-0.0084
k14	0.9918	0.0002	0.9799	0.0004	0.9611	0.0006	0.9264	0.0004	0.8701	-0.0012	0.8019	-0.0044	0.7361	-0.0083
k15	0.9887	0.0000	0.9776	-0.0002	0.9646	-0.0005	0.9443	-0.0015	0.9169	-0.0029	0.8888	-0.0049	0.8673	-0.0063
k16	0.9967	0.0001	0.9911	0.0003	0.9753	0.0006	0.9363	0.0011	0.8557	0.0005	0.7283	-0.0034	0.5750	-0.0123
k20	0.9932	0.0001	0.9826	0.0001	0.9637	-0.0001	0.9281	-0.0014	0.8704	-0.0046	0.7942	-0.0102	0.7149	-0.0162
k21	0.9909	-0.0001	0.9805	-0.0003	0.9645	-0.0007	0.9393	-0.0017	0.9069	-0.0033	0.8707	-0.0050	0.8339	-0.0073
k22	0.9907	0.0000	0.9804	0.0000	0.9677	-0.0001	0.9433	-0.0003	0.8989	-0.0009	0.8316	-0.0029	0.7461	-0.0078
k23	0.9908	-0.0002	0.9687	-0.0007	0.9367	-0.0018	0.9121	-0.0035	0.8840	-0.0055	0.8510	-0.0079	0.8090	-0.0101
k24	0.9945	0.0000	0.9820	-0.0001	0.9431	-0.0004	0.8858	-0.0015	0.8310	-0.0038	0.7617	-0.0077	0.6729	-0.0136
k04	0.9913	0.0001	0.9741	0.0000	0.9555	0.0000	0.9282	0.0002	0.8788	-0.0003	0.8172	-0.0023	0.7594	-0.0041
k09	0.9889	0.0000	0.9739	-0.0001	0.9632	-0.0001	0.9453	-0.0003	0.9128	-0.0011	0.8726	-0.0033	0.8375	-0.0052
k10	0.9912	0.0000	0.9701	0.0001	0.9484	0.0001	0.9269	0.0001	0.8918	-0.0003	0.8395	-0.0024	0.7723	-0.0082
k17	0.9913	-0.0001	0.9765	-0.0002	0.9562	-0.0006	0.9225	-0.0017	0.8716	-0.0040	0.8097	-0.0076	0.7471	-0.0108
k18	0.9877	-0.0004	0.9705	-0.0010	0.9575	-0.0017	0.9441	-0.0026	0.9286	-0.0037	0.9107	-0.0046	0.8900	-0.0048
k19	0.9928	0.0002	0.9846	0.0003	0.9718	0.0002	0.9471	-0.0004	0.9041	-0.0024	0.8395	-0.0069	0.7560	-0.0139
RMS	0.9915	**0.0002**	0.9781	**0.0004**	0.9591	**0.0009**	0.9321	**0.0016**	0.8909	**0.0028**	0.8360	**0.0052**	0.7727	**0.0091**

Codec noise / low-res images

TABLE II. ACCURACY OF THE PROPOSED METHOD FOR COMPUTING SSIM: H.264 ENCODING/DECODING, 384X256 IMAGES.

File	QP=17		QP=22		QP=27		QP=32		QP=37		QP=42		QP=47	
	SSIM	Delta	SSIM	Delta	SSIM	Delta	SSIM	Delta	SSIM	Delta	SSIM	Delta	SSIM	Delta
k01	0.99698	0.00037	0.99194	0.00097	0.97784	0.00241	0.94324	0.00516	0.86771	0.00803	0.74109	0.00880	0.57732	0.00272
k02	0.99501	0.00008	0.98533	0.00023	0.95178	0.00078	0.87002	0.00155	0.75560	0.00090	0.64300	-0.00305	0.56365	-0.00703
k03	0.99188	0.00019	0.98242	0.00052	0.96545	0.00115	0.93660	0.00200	0.89493	0.00280	0.84024	0.00155	0.78627	-0.00283
k05	0.99808	0.00015	0.99444	0.00043	0.98361	0.00114	0.95473	0.00263	0.89125	0.00511	0.77406	0.00763	0.61517	0.00595
k06	0.99536	0.00015	0.98865	0.00034	0.96971	0.00083	0.92473	0.00164	0.84128	0.00247	0.71864	0.00087	0.61210	0.00010
k07	0.99452	0.00031	0.99010	0.00059	0.98075	0.00107	0.96086	0.00157	0.92252	0.00145	0.85035	-0.00080	0.74457	-0.00555
k08	0.99737	0.00038	0.99269	0.00095	0.98136	0.00176	0.95571	0.00298	0.91088	0.00426	0.83130	0.00527	0.69829	0.00666
k11	0.99482	0.00028	0.98655	0.00067	0.96532	0.00127	0.91783	0.00214	0.84478	0.00142	0.76196	-0.00031	0.66883	-0.00503
k12	0.99110	0.00020	0.97866	0.00042	0.95682	0.00083	0.92331	0.00111	0.89056	0.00117	0.85733	0.00099	0.81379	-0.00187
k13	0.99805	0.00007	0.99442	0.00019	0.98139	0.00053	0.93923	0.00123	0.84021	0.00153	0.67614	-0.00186	0.49508	-0.00777
k14	0.99661	0.00010	0.98994	0.00028	0.97108	0.00072	0.92420	0.00131	0.83555	0.00043	0.71927	-0.00351	0.58296	-0.01225
k15	0.99275	0.00017	0.98407	0.00026	0.96894	0.00068	0.94086	0.00142	0.90375	0.00185	0.85833	0.00106	0.80702	-0.00049
k16	0.99331	-0.00004	0.98418	-0.00005	0.96285	-0.00011	0.91732	-0.00047	0.84069	-0.00135	0.74781	-0.00335	0.65946	-0.00897
k20	0.99266	0.00012	0.98866	0.00020	0.97924	0.00030	0.95427	0.00047	0.90503	-0.00029	0.85874	-0.00133	0.81152	-0.00237
k21	0.99262	0.00007	0.98690	0.00025	0.97717	0.00053	0.95577	0.00103	0.91382	0.00182	0.84091	0.00163	0.74318	-0.00203
k22	0.99415	0.00006	0.98579	0.00012	0.96452	0.00026	0.91378	-0.00016	0.82362	-0.00229	0.71706	-0.00778	0.62065	-0.01467
k23	0.99122	-0.00056	0.98422	-0.00092	0.97129	-0.00162	0.94555	-0.00291	0.90752	-0.00497	0.86277	-0.00770	0.81030	-0.00943
k24	0.99620	0.00024	0.99028	0.00054	0.97486	0.00113	0.93799	0.00191	0.85555	0.00213	0.73588	-0.00039	0.58894	-0.01315
k04	0.99329	-0.00016	0.98424	-0.00042	0.96570	-0.00098	0.92860	-0.00179	0.86661	-0.00295	0.78177	-0.00573	0.69493	-0.00935
k09	0.99097	0.00022	0.98611	0.00041	0.97816	0.00075	0.96182	0.00125	0.92696	0.00132	0.86711	0.00144	0.78502	-0.00182
k10	0.99198	0.00007	0.98368	0.00009	0.96899	0.00004	0.94139	-0.00020	0.89478	-0.00067	0.81985	-0.00315	0.72914	-0.00769
k17	0.99421	0.00013	0.98443	0.00031	0.96673	0.00036	0.93235	0.00020	0.87392	-0.00140	0.79433	-0.00467	0.69088	-0.01134
k18	0.99589	0.00012	0.98830	0.00032	0.97295	0.00073	0.93719	0.00127	0.85561	0.00123	0.70694	-0.00226	0.54630	-0.01039
k19	0.99318	-0.00006	0.98357	-0.00018	0.96557	-0.00039	0.92249	-0.00078	0.84125	-0.00251	0.77152	-0.00393	0.70107	-0.00336
RMS	0.99426	**0.00022**	0.98707	**0.00048**	0.97095	**0.00100**	0.93520	**0.00189**	0.87188	**0.00285**	0.78502	**0.00417**	0.68746	**0.00758**

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Gaussian blur

TABLE III. ACCURACY OF THE PROPOSED METHOD FOR COMPUTING SSIM: GAUSSIAN BLUR, 1536X1024 IMAGES.

File	sigma=0.5		sigma=0.7		sigma=1		sigma=3		sigma=5		sigma=10		sigma=15	
	SSIM	Delta	SSIM	Delta	SSIM	Delta	SSIM	Delta	SSIM	Delta	SSIM	Delta	SSIM	Delta
k01	0.98720	0.00048	0.94682	0.00165	0.87318	0.00211	0.55327	-0.01726	0.47744	-0.03227	0.44126	-0.02791	0.43598	-0.02557
k02	0.98573	0.00008	0.94714	0.00007	0.88935	-0.00093	0.68282	-0.01827	0.64281	-0.01764	0.62244	-0.01382	0.61872	-0.01157
k03	0.99402	0.00002	0.97981	-0.00001	0.95905	-0.00053	0.86614	-0.01306	0.83851	-0.01443	0.82062	-0.01167	0.81688	-0.00989
k05	0.99394	0.00013	0.97588	0.00023	0.93818	-0.00074	0.66111	-0.02633	0.55650	-0.03670	0.49014	-0.02812	0.47903	-0.02371
k06	0.99006	0.00021	0.95896	0.00061	0.90340	0.00021	0.65718	-0.01601	0.60000	-0.02167	0.57249	-0.01801	0.56856	-0.01613
k07	0.99494	-0.00008	0.98365	-0.00039	0.96785	-0.00138	0.87293	-0.01663	0.80660	-0.01403	0.74749	-0.00715	0.73933	-0.00321
k08	0.98452	0.00052	0.93801	0.00147	0.86230	0.00079	0.54442	-0.02261	0.45929	-0.04163	0.40624	-0.03386	0.39683	-0.02919
k11	0.98950	0.00015	0.95933	0.00030	0.90901	-0.00057	0.70088	-0.01919	0.64987	-0.02343	0.61996	-0.01886	0.61386	-0.01632
k12	0.99447	0.00001	0.98034	-0.00010	0.95877	-0.00091	0.86787	-0.01407	0.84303	-0.01567	0.82811	-0.01359	0.82542	-0.01226
k13	0.98144	0.00029	0.92398	0.00077	0.82716	-0.00023	0.46356	-0.01772	0.38538	-0.02603	0.34533	-0.01965	0.33909	-0.01612
k14	0.99087	-0.00001	0.96487	-0.00034	0.91966	-0.00214	0.69399	-0.02678	0.62321	-0.02971	0.57784	-0.02213	0.56904	-0.01755
k15	0.99268	-0.00004	0.97245	-0.00028	0.94195	-0.00129	0.82807	-0.01410	0.79545	-0.01633	0.76964	-0.01186	0.76280	-0.00892
k16	0.99004	0.00012	0.96061	0.00036	0.91245	-0.00006	0.73105	-0.01430	0.69283	-0.01578	0.67404	-0.01293	0.67084	-0.01125
k20	0.99160	0.00004	0.97006	-0.00001	0.93853	-0.00067	0.82382	-0.01119	0.79265	-0.01426	0.77086	-0.01084	0.76631	-0.00903
k21	0.98786	0.00009	0.95579	0.00006	0.90823	-0.00114	0.72230	-0.01587	0.67120	-0.02187	0.64053	-0.01709	0.63514	-0.01424
k22	0.99040	0.00002	0.96446	-0.00006	0.92320	-0.00102	0.74711	-0.01904	0.69702	-0.01675	0.66932	-0.01154	0.66455	-0.00843
k23	0.99399	-0.00016	0.98079	-0.00059	0.96404	-0.00143	0.89616	-0.01090	0.87084	-0.00964	0.85306	-0.00616	0.84902	-0.00367
k24	0.99288	0.00000	0.97165	-0.00029	0.93133	-0.00191	0.70978	-0.02570	0.64305	-0.02919	0.60436	-0.02300	0.59750	-0.01931
k04	0.99146	-0.00007	0.96870	-0.00040	0.93370	-0.00161	0.78886	-0.01777	0.74753	-0.01971	0.71973	-0.01498	0.71366	-0.01167
k09	0.98837	0.00007	0.96191	0.00007	0.93092	-0.00059	0.82522	-0.01349	0.78759	-0.01683	0.76389	-0.01334	0.76003	-0.01129
k10	0.98907	0.00002	0.96433	-0.00015	0.93486	-0.00116	0.82576	-0.01746	0.78687	-0.01906	0.76038	-0.01491	0.75543	-0.01235
k17	0.98826	-0.00022	0.96069	-0.00095	0.92650	-0.00267	0.80626	-0.01901	0.76054	-0.01877	0.72108	-0.01130	0.71121	-0.00627
k18	0.98254	0.00003	0.93664	-0.00016	0.87364	-0.00162	0.63151	-0.02020	0.55425	-0.01938	0.50747	-0.01122	0.49986	-0.00686
k19	0.98872	0.00011	0.95763	0.00024	0.90834	-0.00040	0.71598	-0.01373	0.66602	-0.02025	0.63962	-0.01697	0.63520	-0.01502
RMS	0.98978	**0.00018**	0.96197	**0.00058**	0.91876	**0.00127**	0.74227	**0.01805**	0.69289	**0.02260**	0.66235	**0.01758**	0.65678	**0.01476**

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Additive noise

TABLE IV. ACCURACY OF THE PROPOSED METHOD FOR COMPUTING SSIM: NOISE, 1536X1024 IMAGES.

File	p=0.00001		p=0.0005		p=0.001		p=0.005		p=0.01		p=0.05		p=0.1	
	SSIM	Delta	SSIM	Delta	SSIM	Delta	SSIM	Delta	SSIM	Delta	SSIM	Delta	SSIM	Delta
k01	0.99903	0.00001	0.99429	0.00016	0.98857	0.00039	0.94685	0.00179	0.90066	0.00308	0.66301	0.00715	0.51428	0.00692
k02	0.99760	0.00001	0.98705	0.00002	0.97412	0.00003	0.87685	-0.00020	0.77454	-0.00089	0.35328	-0.00377	0.19618	-0.00228
k03	0.99837	0.00001	0.99119	0.00001	0.98273	0.00000	0.91882	-0.00026	0.85090	-0.00082	0.54850	-0.00307	0.40787	-0.00254
k05	0.99842	0.00006	0.99118	0.00025	0.98250	0.00049	0.91927	0.00181	0.85094	0.00298	0.53878	0.00301	0.38195	0.00123
k06	0.99826	0.00001	0.99119	0.00007	0.98259	0.00016	0.91792	0.00045	0.84912	0.00064	0.55184	0.00166	0.40942	0.00264
k07	0.99865	-0.00002	0.99349	-0.00013	0.98663	-0.00016	0.93714	-0.00113	0.88169	-0.00216	0.60637	-0.00724	0.45282	-0.00830
k08	0.99896	0.00002	0.99328	0.00019	0.98633	0.00038	0.93630	0.00179	0.88357	0.00291	0.62646	0.00496	0.47785	0.00392
k11	0.99867	0.00002	0.99314	0.00007	0.98642	0.00012	0.93822	0.00043	0.88717	0.00061	0.66258	0.00104	0.54117	0.00104
k12	0.99751	0.00000	0.98667	-0.00001	0.97341	0.00000	0.87508	-0.00028	0.77109	-0.00092	0.34640	-0.00354	0.19323	-0.00192
k13	0.99898	0.00002	0.99486	0.00006	0.98993	0.00014	0.95029	0.00065	0.90515	0.00102	0.66921	0.00215	0.51826	0.00206
k14	0.99835	-0.00001	0.99190	0.00001	0.98361	0.00005	0.92031	0.00020	0.85224	0.00008	0.54490	-0.00125	0.39510	-0.00142
k15	0.99692	-0.00003	0.98302	-0.00014	0.96643	-0.00027	0.84714	-0.00163	0.72546	-0.00347	0.29058	-0.00677	0.16568	-0.00320
k16	0.99860	0.00000	0.99306	0.00003	0.98600	0.00005	0.93347	0.00011	0.87764	0.00007	0.61034	-0.00076	0.46907	-0.00058
k20	0.99688	-0.00003	0.98238	-0.00016	0.96519	-0.00031	0.84397	-0.00179	0.72177	-0.00360	0.30715	-0.00476	0.19120	-0.00147
k21	0.99884	0.00001	0.99381	0.00003	0.98732	0.00010	0.94062	0.00049	0.88761	0.00066	0.61701	-0.00006	0.45622	-0.00111
k22	0.99861	0.00000	0.99279	0.00001	0.98596	0.00000	0.93262	-0.00023	0.87467	-0.00061	0.58993	-0.00272	0.43961	-0.00260
k23	0.99810	-0.00001	0.98975	-0.00013	0.98002	-0.00027	0.90565	-0.00137	0.82478	-0.00265	0.47568	-0.00734	0.32576	-0.00627
k24	0.99842	0.00003	0.99199	0.00007	0.98442	0.00012	0.92877	0.00050	0.86804	0.00065	0.58666	0.00007	0.43818	-0.00065
k04	0.99837	0.00000	0.99075	-0.00003	0.98146	-0.00005	0.91511	-0.00056	0.84378	-0.00128	0.52395	-0.00479	0.37573	-0.00494
k09	0.99852	0.00001	0.99229	0.00004	0.98434	0.00007	0.92621	0.00013	0.86374	0.00007	0.56167	-0.00171	0.40668	-0.00218
k10	0.99875	0.00001	0.99389	-0.00001	0.98791	0.00003	0.94274	0.00005	0.89303	-0.00014	0.64885	-0.00223	0.51264	-0.00324
k17	0.99745	-0.00003	0.98754	-0.00018	0.97569	-0.00036	0.88805	-0.00184	0.79812	-0.00363	0.44390	-0.00773	0.30673	-0.00594
k18	0.99816	-0.00001	0.99028	-0.00002	0.97994	-0.00009	0.90481	-0.00068	0.82380	-0.00151	0.46808	-0.00462	0.30914	-0.00354
k19	0.99885	0.00002	0.99384	0.00013	0.98743	0.00026	0.94086	0.00109	0.88889	0.00172	0.62521	0.00182	0.47511	0.00021
RMS	0.99830	**0.00002**	0.99099	**0.00011**	0.98206	**0.00022**	0.91659	**0.00103**	0.84732	**0.00193**	0.54743	**0.00423**	0.40514	**0.00360**

Consequences & Applications

Main topics:

- Relationship between SSIM and other objective metrics
- Boosting SSIM scores by LF/HF noise-shaping and pre-filtering
- More sophisticated models based on SSIM

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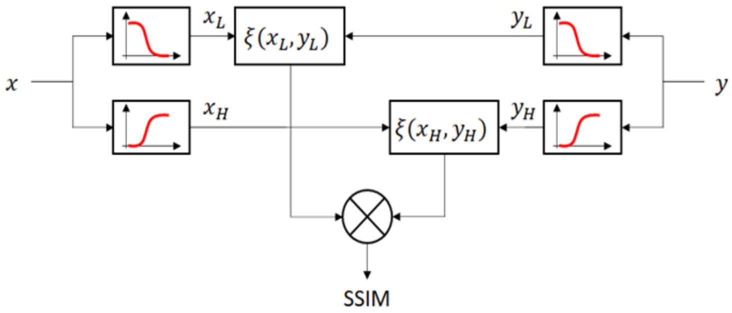
Relation to other objective metrics

Symmetric 2-band definition

- Full formula: $SSIM(x,y) \approx \xi(x_L,y_L)\cdot\xi(x_H,y_H)$
  where: $\xi(x,y) = \frac{2\boldsymbol{E}[xy]+C}{\boldsymbol{E}[x^2]+\boldsymbol{E}[y^2]+C}$



Connection to MSE:

- Let's expand: $\frac{1}{\xi(x,y)} = \frac{\boldsymbol{E}[x^2]+\boldsymbol{E}[(x+(y-x))^2]}{2\boldsymbol{E}[xy]} = 1 + \frac{\boldsymbol{E}[(y-x)^2]}{2\boldsymbol{E}[xy]}$
- Consequently: $\frac{1-\xi(x,y)}{\xi(x,y)} = \frac{MSE(x,y)}{2\boldsymbol{E}[xy]}$ - refinement of result of [Hoare & Zhou, 2010]
- Problem: one still needs to know covariance $\boldsymbol{E}[xy]$ to connect to MSE/PSNR!

Connection to SNR:

- Let's expand it differently : $\xi(x,y) = 1 - \frac{\boldsymbol{E}[(y-x)^2]}{\boldsymbol{E}[x^2]+\boldsymbol{E}[y^2]}$
- Consequently: $\frac{1}{1-\xi(x,y)} = SNR(x,y) + SNR(y,x)$
- Full formula for patch-level SSIM: $SSIM(x,y) \sim \left(1 - \frac{1}{SNR(x_L,y_L)+SNR(y_L,x_L)}\right) \times \left(1 - \frac{1}{SNR(x_H,y_H)+ SNR(y_H,x_H)}\right)$

- NB: SSIM can be computed as a composite of SNR values computed for low-pass and high-pass images!
- So the connection to other objective metrics such as SNR or PSNR exits, but it is not that simple!

Boosting SSIM scores by pre-filtering

The game of balancing errors:

- Idea: distribute the errors between LF and HF bands such that $SSIM(x,y) \approx \xi(x_L, y_L) \cdot \xi(x_H, y_H)$ is maximal.
- If errors are tradeable, then one can show that: $\max_{\delta:\, \xi_L = \xi_0 + \delta,\, \xi_H = \xi_0 - \delta} SSIM(\xi_L, \xi_H) \Rightarrow \delta = 0 \Rightarrow \xi_L = \xi_H.$

Simple practical solution: apply unsharp mask filter!

- Effects as visible in uncompressed image:

Original image

After unsharp mask

The same images after H.264/Intra encoding with QP=42:

Original H.264 encoded QP=42
MS-SSIM=0.9275

After unsharp mask H.264 QP=42
MS-SSIM=0.9390

Credit: Luc Trudeau : https://medium.com/ @luc.trudeau/a-pre-processing-filter-for-ssim-a60ce91f3374

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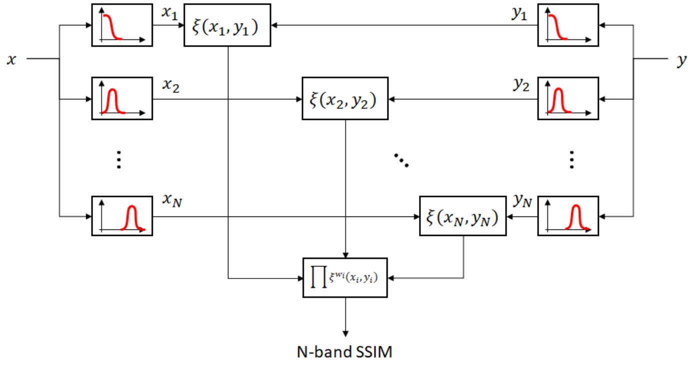
Back to error visibility?

Extend SSIM model to N bands:

- N-band SSIM:

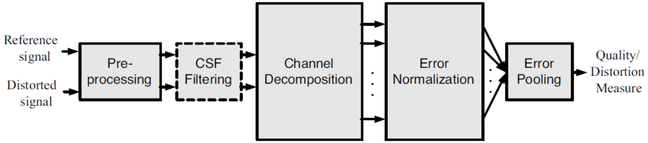
$$SSIM(x,y) = \prod_{i=1}^{N} \left(\xi(x_i, y_i)\right)^{w_i}, \qquad \xi(x,y) = \frac{2\boldsymbol{E}[xy] + C}{\boldsymbol{E}[x^2] + \boldsymbol{E}[y^2] + C}$$

- where:
  - x_i, y_i - are band-filtered versions of input images,
  - w_i are perceptual weights applied to band-level scores,
  - C is a small constant.

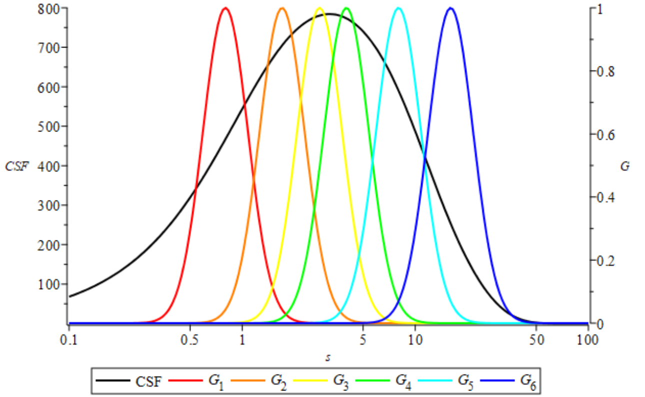


Natural extension of SSIM:

- Uses the same atomic patch-level measures as SSIM
- Subbands that can be defined in angular units, aligned with CSF model (unlike MS-SSIM, which operates at fixed pixel-level scales)
- Connects well to perceptual science: Barten CSF modes, SQRI, Sarnoff JND model, VDP, etc. Back to the starting point for SSIM!



CSF and 6 filters centered @ 0.8, 1.7, 2.8, 4, and 8 cpd



CONCLUSIONS

Summary & Outlook

New model for computing SSIM is introduced

- Uses 2-band decomposition + symmetric processing in each band

It works

- Matches the original SSIM model under
  - Codec noise
  - Resolution changes
  - Gaussian blur
  - Additive noise

Utilities

- Methodological
- Enables encoder and filter optimizations
- Explains connection to other objective metrics (SNR, PSNR, MSE)
- Brings us back to perceptual models!

Next steps

- N-band SSIM – this promises to be a lot of fun!
- Parametric quality models – returnable to different screen sizes, SDR/HDR, etc. Much needed for multi-screen!

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THANK YOU